Computational Tools for Macroeconomics using MATLAB

Week 11 – Macroeconometrics: OLS & VARs

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Learning Outcomes

By the end of this week, you will be able to:

- **Import and Transform Data**: Clean FRED data (GDP, Inflation, Interest Rates) and compute growth rates.
- **Run OLS Regressions**: Estimate simple relationships like the Phillips Curve.
- **Estimate VARs**: Build and estimate a Vector Autoregression (VAR) model from scratch.
- **Identify Shocks**: Understand and implement the Cholesky decomposition for structural identification.
- **Compute IRFs**: Generate and plot Impulse Response Functions to analyze economic dynamics.

The Roadmap

1. **Data Handling**: Getting the data ready for analysis.
2. **The Phillips Curve**: A simple OLS example.
3. **Vector Autoregressions (VARs)**: Theory and Estimation.
4. **Identification**: From Reduced Form to Structural Shocks.
5. **Impulse Responses**: Tracing the effects of shocks.
6. **Challenge**: Bivariate VAR (GDP & Inflation).

Data Sources: FRED

- *week11_get_data.m*

We will use US quarterly data from the Federal Reserve Economic Data (FRED):

- **Real GDP** (`GDPC1`): Measure of economic activity.
- **GDP Deflator** (`GDPDEF`): Measure of aggregate price level.
- **Federal Funds Rate** (`FEDFUNDS`): Policy interest rate.

- *week11.m*

Transformations for Stationarity:

- GDP → Growth Rate: $400 \times \Delta \ln(Y_t)$
- Price Level → Inflation: $400 \times \Delta \ln(P_t)$
- Interest Rate → Levels (already in %)

MATLAB: Data Transformation

```
% Annualized Quarterly Growth Rate
% 400 * (ln(Y_t) - ln(Y_{t-1}))
gdp_growth = [NaN; 400 * diff(log(gdp_level))];

% Annualized Inflation Rate
inflation  = [NaN; 400 * diff(log(deflator))];

% Align data (remove the first NaN observation)
data_raw   = [gdp_growth, inflation, ffr];
data_clean = data_raw(2:end, :);
```

The Simple Phillips Curve

Consider a simple relationship between Inflation (π_t) and real activity (y_t):

$$\pi_t = \alpha + \beta y_t + u_t$$

We can estimate this using Ordinary Least Squares (OLS).

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Warning: Endogeneity!

- This regression likely suffers from **simultaneity bias**.
- π_t and y_t are determined jointly in equilibrium.
- OLS captures correlation, not necessarily causality.
- *This motivates the use of VARs and Structural Identification.*

MATLAB: Manual OLS

```
Y_ols = data_clean(:, 2); % Inflation
X_ols = [ones(length(Y_ols),1), data_clean(:, 1)];

% Manual OLS Estimator: beta_hat = (X'X)^-1 X'Y
beta_hat = (X_ols' * X_ols) \ (X_ols' * Y_ols);

fprintf('Constant: %.4f\n', beta_hat(1));
fprintf('Slope: %.4f\n', beta_hat(2));
```

Check your results against the built-in function:

```
mdl = fitlm(data_clean(:,1), data_clean(:,2));
```

What is a VAR?

A **Vector Autoregression (VAR)** generalizes the univariate AR model to multiple variables.

Structure of a VAR(1) with 2 variables (y_t, π_t):

$$\begin{pmatrix} y_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ \pi_{t-1} \end{pmatrix} + \begin{pmatrix} u_{1,t} \\ u_{2,t} \end{pmatrix}$$

- Every variable depends on its own lags AND the lags of all other variables.
- u_t are **reduced-form residuals** (correlated).

Estimation: Equation by Equation

Since the regressors (lags) are the same for each equation, OLS is efficient.

```
% Construct Lagged Matrices
Y_lhs = data(2:end, :);        % t = 2...T
X_rhs = [ones(T-1, 1), data(1:end-1, :)]; % Const + Y_{t-1}

% Estimate Coefficients
B = (X_rhs' * X_rhs) \ (X_rhs' * Y_lhs);

% Extract Transition Matrix A1
A1 = B(2:end, :)';
```

The Identification Problem

The estimated residuals u_t are correlated:

$$\Sigma_u = E[u_t u_t'] \neq I$$

We want to recover **structural shocks** ϵ_t which are orthogonal ($E[\epsilon_t \epsilon_t'] = I$).

$$u_t = P\epsilon_t \implies \Sigma_u = PP'$$

Cholesky Decomposition: Assume P is lower triangular.

- Ordering matters! ($y_t \rightarrow \pi_t$)
- Variable 1 (y_t) does not respond to Variable 2 (π_t) contemporaneously.

MATLAB: Identification

```
% 1. Calculate Residuals
residuals = Y_lhs - X_rhs * B;
Sigma = cov(residuals);

% 2. Cholesky Decomposition (Lower Triangular)
P = chol(Sigma, 'lower');

% Structural Shocks
shock_gdp = P(:, 1); % Shock to first variable
shock_inf = P(:, 2); % Shock to second variable
```

Impulse Response Functions (IRFs)

How does the system respond to a shock ϵ_j over time?

$$X_0 = P\epsilon_j \quad \text{(Impact)}$$

$$X_1 = A_1 X_0$$

$$X_2 = A_1 X_1 = A_1^2 X_0$$

$$\vdots$$

$$X_h = A_1^h P\epsilon_j$$

MATLAB: Computing IRFs

```
H = 16; % Horizon
irf = zeros(2, H);

% Initial Impact
irf(:, 1) = P(:, 1); % Response to Shock 1

% Iterate Forward
for h = 2:H
    irf(:, h) = A1 * irf(:, h-1);
end
```

In-Class Challenge

Goal: Build a Bivariate VAR for GDP Growth and Inflation.

1. Load `US_macro_data.csv`.
2. Estimate the VAR(1) coefficients.
3. Identify shocks using Cholesky (*Growth* $\rightarrow$ *Inflation*).
4. Plot the Impulse Responses.

Open `week11_challenge_starter.m` to begin!

Homework: Monetary Policy Shocks

Task: Extend the model to a 3-variable VAR.

$$X_t = [\text{GDP Growth, Inflation, Interest Rate}]'$$

- Ordering: *GDP* → *Inflation* → *R*.
- Assumption: Interest rates respond to output and prices instantly, but the macroeconomy responds to rates with a lag.
- **Analyze**: Does a hike in *R* cause a recession (drop in *GDP*)? Does inflation fall?